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CLOSE BINARY STARS

Otto Struve and Su-Shu Huang

1. INTRODUCTION

Although the spectroscopic peculiarities of some individual eclipsing binaries, such as β Lyrae, etc., have been known for a long time, a systematic study of the problem of close binaries has only recently been undertaken. Some important astrophysical problems resulting from the spectroscopic study of eclipsing binaries were proposed in a series of papers in the *Astrophysical Journal* (1) and have since then become the main interest of later investigations by several authors, including J. Sahade, W. A. Hiltner, C. U. Cesco, and others. In reviewing these problems we find that some of them have been solved while others have opened up new fields of study. These investigations have greatly helped our understanding of the problem of stellar evolution in general.

It is impractical in this brief review to cover all the spectroscopic problems connected with close binaries; we have, therefore, selected only those topics which are associated with the problem of gaseous streams in close binary systems.

As the motion of gaseous streams in a binary system can be approximately treated as a restricted three-body problem (2), a working knowledge of the problem will be useful for interpreting the results observed in various systems. Section 2 summarizes a certain phase of this problem which is useful for the present purpose.

The gaseous stream associated with a binary system manifests itself in the form of emission lines, of absorption lines which are seen superposed over the normal stellar absorption lines, and in distortions of the velocity-curves produced by lines which cannot be seen as distinct features on top of the stellar lines. Thus, in Sections 3 and 4 we discuss the manner in which we correct the velocity-curve which has been distorted by gaseous streams, and in Section 5 we outline the problem of emission lines observed in some of the systems.

Among the close binaries in which streams of gas exist, β Lyrae is the most remarkable one. It has been extensively studied, but it is not yet fully understood. We have therefore included a whole section (6) describing this remarkable star.

In the last two sections (7 and 8), the problem of evolution of the component stars of some close binary systems is discussed in the light of the observations

of sub-giant components of Algol-type variables (3). We have also outlined a hypothesis which predicts a sequence of unstable stars as a result of the loss of mass suffered by the components in close binary systems (4).

2. The Restricted Problem of Three Bodies

The restricted three-body problem discusses the motion of an infinitesimal body which is attracted gravitationally by two finite bodies revolving in circles around the centre of mass. In other words, the infinitesimal body has such a small mass that its gravitational force on the two finite bodies can be neglected.

Following F. R. Moulton (2) we let, by proper choice of units, the sum of the masses of the two finite bodies and the distance between them be unity. Therefore, we can represent the individual masses of the two finite bodies by $1-\mu$ and μ. It is understood in the following discussion that $\mu \leqslant \frac{1}{2}$. Furthermore, we choose the unit of time such that the gravitational constant is unity. As a consequence of the way the units have to be chosen, one can easily prove that the angular velocity of the finite bodies in their circular orbit is unity also.

We now refer the motion of the bodies to a rectangular coordinate system which has its origin at the centre of mass of the two finite bodies, its x-axis always coinciding with the line joining them, and its z-axis being perpendicular to the plane of their motion. The equation of motion of the third body can then be written down; it admits the following integral

$$V^2 = -2\Omega - C, \tag{1}$$

where

$$\Omega = -\tfrac{1}{2}(x^2+y^2) - \frac{(1-\mu)}{r_1} - \frac{\mu}{r_2} \tag{2}$$

defines the potential field produced by the two finite bodies in the rotating frame of reference, V is the velocity of the infinitesimal body, and C is a constant of integration. In equation (2), r_1 and r_2 represent the distances of the infinitesimal body from the two finite ones. When the constant of integration, C, has been determined by the initial conditions, equation (1) gives the velocity with which the infinitesimal body moves at any permissible point. For a given value of C, equation (1) together with equation (2) defines a surface for each value of V. In particular, if V is put equal to zero in this equation, it will define a surface, called the zero relative-velocity surface, at which the velocity will be zero. Hence on one side of the zero-velocity surface the velocity will be real and on the other side imaginary. In other words, it is possible for the infinitesimal body to move only on one side and impossible for it to move on the other. Consequently, once the numerical value C (or a definite initial condition) is given, the part of space to which the infinitesimal body is restricted is determined by the zero-velocity surface, although this does not give the actual orbit.

According to the definition, the zero-velocity surface is given by

$$x^2+y^2+\frac{2(1-\mu)}{r_1}+\frac{2\mu}{r_2} = C. \tag{3}$$

It can be seen from equation (3) that the zero-velocity surfaces are symmetrical with respect to the xy and xz planes and, when $\mu = \frac{1}{2}$, with respect to the yz plane,

too. The exact shape of the surface depends upon the value of C. For large C it consists of two separate ovals closed around each of the two finite bodies. For the left-hand side of equation (3) can be large either when r_1 (or r_2) is small or x^2+y^2 is large, the latter case being of no interest at present. Hence large values of C correspond to zero-velocity surfaces differing only slightly from spheres. The greater the value C, the more nearly the surface approaches a sphere. With diminishing value of C, the ovals will tend to become more and more elongated, until, for a certain critical value of C, the two ovals will touch each other at a single point on the x-axis to form a dumb-bell-like configuration. The point of contact on the x-axis is called the first double point, L_1. The zero-velocity surface passing through L_1, sometimes known as the inner contact surface, has been calculated recently by G. P. Kuiper (**5**) and Z. Kopal (**6**) for different values of the mass ratio (or μ). For still smaller values of C the connecting part of the dumb-bell opens up, and a single zero-velocity surface encloses both bodies. As C decreases the dumb-bell increases in size and finally the surface opens up first at L_2 on the right, and then at L_3 on the left side of the system, since further decrease of C makes the surface move away from the x-axis. This is illustrated in Fig. 1. For example, the surface passing through L_2 corresponds to $C=3{\cdot}520$, hence surfaces corresponding to values of C smaller than $3{\cdot}520$ do not intercept the x-axis on the right side of the system. Two such surfaces are shown in the figure with C equal to $3{\cdot}380$ and $3{\cdot}300$. Similarly, surfaces corresponding to values of C smaller than $3{\cdot}380$, which corresponds to the surface passing through L_3, do not cross the x-axis, as, for example, one with C equal to $3{\cdot}300$. Thus, we obtain two more double points on the x axis. The zero-velocity surfaces passing through each of the two double points are sometimes called the outer contact surfaces. Altogether we have three double points on the x-axis. There are two more double points which appear, as the surfaces vanish from the xy plane, at the two points making equilateral triangles with the finite bodies.

Kuiper (**5**) has computed some zero-velocity surfaces for the case $\mu=0{\cdot}4$, corresponding to a mass ratio of $1{\cdot}5$. Fig. 1 shows his result together with some of our own computations. It represents the intersection of six zero-velocity surfaces with the xy plane including three critical surfaces which pass through the three double points on the x-axis.

The double points are of importance because the conditions of obtaining double points are also the conditions that the components of acceleration in the rotating frame of reference shall vanish. Consequently, if the infinitesimal body is placed at a double point with zero relative velocity, its coordinates will identically fulfil the differential equations of motion and it will remain for ever relatively at rest, unless disturbed by forces exterior to the system. These are particular solutions of the problem of three bodies, and are special cases of the Lagrangian solution (**2**). For this reason the double points are also known as the Lagrangian points. The three double points on the x-axis and the two double points forming equilateral triangles with the two finite bodies represent the so-called straight-line and equilateral triangle solutions. In a Galilean (i.e. stationary) frame of reference, they represent the rotation about the centre of mass of three bodies as though fixed to a straight line or fixed to an equilateral triangle. Such motion can be realized by appropriate initial conditions.

The particular solutions we have discussed are all periodic because the bodies resume, after regular intervals of time, the same positions in a Galilean frame attached to the centre of mass of the system. Once a periodic solution is known, many other solutions may be obtained by changing slightly the initial conditions. Let us consider a known periodic solution with the three bodies describing closed orbits with a definite period. Suppose now we vary slightly the initial conditions (e.g. by changing slightly the initial position and velocity of the infinitesimal body). The new orbits may deviate rapidly from the periodic

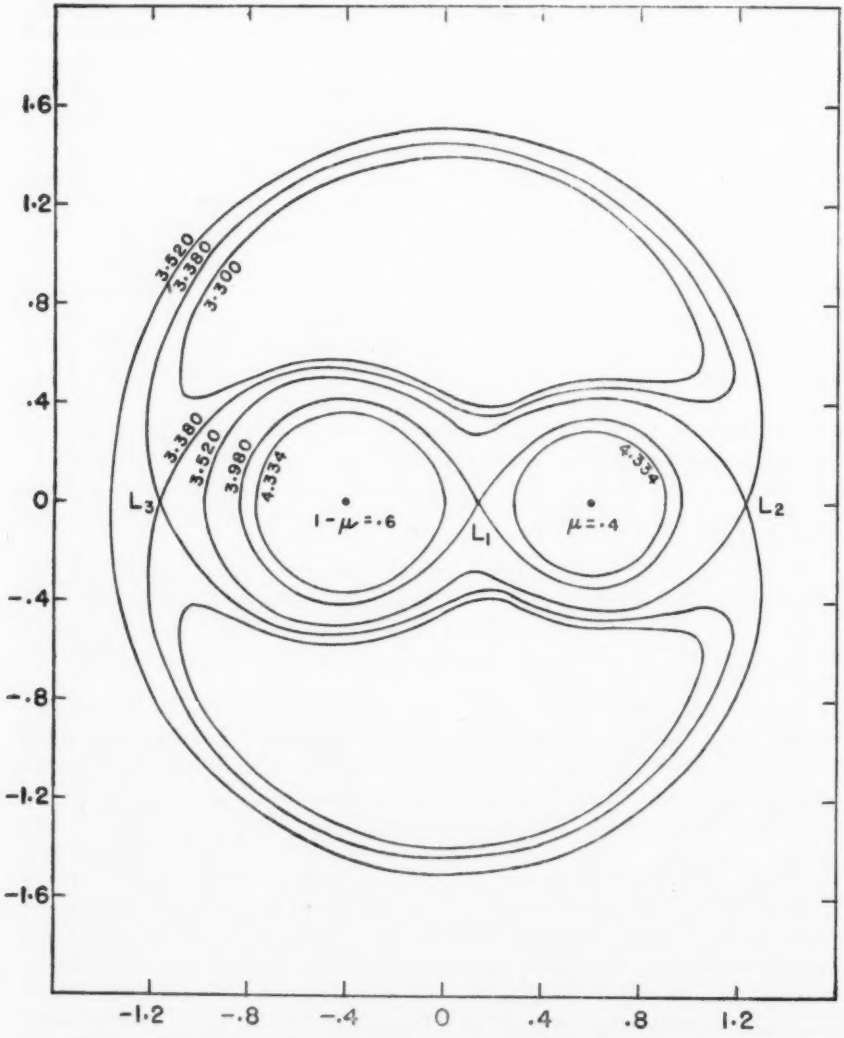


FIG. 1.—*Intersection of the xy-plane and the zero-velocity surfaces associated with two mass points of a mass ratio of 1·5. The value, C, is given for each curve in the figure. L_1, L_2, and L_3 are the three double points on the x-axis.*

ones, or they may never depart very far from the periodic ones. In the former case the periodic solution is called unstable, while in the latter case, it is called stable.

A mathematical study of the stability of the Lagrangian solutions shows that aside from some special cases of initial conditions, the straight-line solution is unstable in general, while the equilateral triangle solution is stable or unstable according as μ is smaller or greater than 0·0385.

Examples of both the straight-line and triangle solutions can be found in the solar system. One of the double points of the Sun–Earth system is exactly opposite to the Sun. Therefore, if a meteor should pass near this point with appropriate velocity, it may stay near it for several revolutions of the Earth around the Sun before it wanders away. If a great number of such objects are swarming around the point at one time, they would appear from the Earth as a hazy patch of light with its centre at the anti-sun. This is actually observed in the form of the Gegenschein (**2**).

We can find an application of the triangle solution in the Trojan family of asteroids, since the family as a whole, together with the planet Jupiter and the Sun, moves as though fixed to the corners of a revolving equilateral triangle.

So far we have treated the stars as mass points. If, however, a star is formed on the Roche model, i.e. its mass is centrally condensed, the previous results about the double points and zero-velocity surfaces can be applied to the study of close binary systems. Thus, the zero-velocity surface is then identical with the equipotential surface in the rotating frame of a binary system built on the Roche model. We label the zero-velocity surface by the value C, which is determined by the initial conditions, while for the potential surface we use as our label the value of the potential, Ω. It is obvious from equations (2) and (3) that

$$C = -2\Omega. \tag{4}$$

Hence the potential increases as the point moves away from the components of a binary system just as in the case of a single star. An important difference, however, is that the surface is no longer spherical. Consequently, if there exist some kinds of prominences (or gaseous ejections) on the surfaces of the components, the gas can escape from the system more easily in some directions than in others. Because of the term $-(x^2+y^2)/2$ in Ω, the potential in the rotating frame drops as we go great distances from the system. Accordingly, along the x-axis, Ω has maxima at both L_2 and L_3, although along the y and z directions Ω has minima at these two points. Similarly at the point L_1, Ω has a maximum along the x-axis but a minimum along either the y or the z directions. Consequently, the three points, L_1, L_2 and L_3, are the saddle points with respect to Ω, and can be regarded as the mountain passes on a highway. L_1 provides a route for transferring mass from one star to the other, while L_2 and L_3 control the flow of mass from the system to the outer space.

Another interesting point we can speculate about in the case of a close binary is that any mass streaming in the system may be trapped in the neighbourhood of the double points just as the Gegenschein and the Trojan family of asteroids are trapped in the solar system (**7**). Since the straight-line solutions are in general unstable, we would not expect a permanent condensation of gases at the double points on the x-axis. Gaseous particles come and go, but the density

there could reach a higher value than elsewhere, exactly as in the Gegenschein. The observations seem to show that concentrations of gas at L_1 and L_2 may exist. The triangle solutions are also unstable unless the masses are so different that $\mu < 0{\cdot}0385$. In the case $\mu < 0{\cdot}0385$, permanent condensations around the double points corresponding to the triangle solution should be expected. The mass ratio of the components of a binary system in which $\mu \leqslant 0{\cdot}0385$ should be 25 or greater. Known systems of such mass ratios are very rare because of observational selection.

The actual streaming of gases in the system is an extremely difficult problem. It requires the solution of the hydrodynamical equations of motion in a two-centre force field. By symmetry conditions one might simplify the equations slightly, but so far no progress has been made in this direction. Consequently our knowledge of gaseous streams in close binary systems has been derived from the observations.

3. Distribution of the Longitude of the Periastron of Spectroscopic Binary Stars

It is well known that the eccentricity, e, of the orbit of a spectroscopic binary and the longitude, ω, of the periastron or of the line of apsides can be determined from the radial velocity-curve. The longitude of the periastron is defined by the angle in the plane of the orbit between the lines of nodes and the periastron, measured from the ascending node to the periastron, in the direction of the motion of the component. Hence the eccentricity determines the shape of the orbit, while the longitude of the periastron determines its orientation in the orbital plane. For example, when the periastron is at the farthest point of the orbit as seen from the Earth, $\omega = 90^\circ$. When the periastron is at the nearest point, $\omega = 270^\circ$. When it is in the plane of the celestial sphere with the star receding, $\omega = 0^\circ$, and when it is in the plane of the celestial sphere with the star approaching, $\omega = 180^\circ$. It therefore becomes obvious that aside from two scale factors, one connected with each coordinate axis, the shape of the velocity-curve of the components of spectroscopic binaries is determined completely by the two orbital elements, e and ω, the two scale factors being determined by the semi-major axis, the period and the inclination of the orbit. Conversely, the two orbital elements can be determined, once the velocity-curve is known. If the velocity-curve should correctly represent the star's motion in its orbit, there would be no doubt about the correctness of the values of e and ω determined from it. On the other hand if we should show that the values of e and ω determined from the velocity-curve are biased, the velocity-curve must have been distorted by some unknown phenomenon. We shall give ample evidence that the velocity-curves of certain binary stars are distorted by gaseous streams in the systems.

Statistical studies of the orbital elements of spectroscopic binaries show an unexpected distribution of ω: there is a decided tendency of the angle ω to be more often between 0° and 90° than in the other quadrants of the circle. The phenomenon, i.e. the non-uniform distribution of ω of spectroscopic binaries, was first pointed out by J. M. Barr (**8**), and is consequently known as the Barr effect. If, as is reasonable to expect, the orbits of the binaries are distributed at random in space, we should predict that except for minor random fluctuations

the distribution of ω would be approximately the same in all four quadrants. There has been a great deal of discussion during the past fifty years regarding the reality of the uneven distribution of the values of ω and of its possible physical significance. Two alternative explanations have been proposed: the one regards the uneven distribution of ω as a result of pure chance and the other takes it as manifestation of a real phenomenon in nature and attempts to explain it.

In 1929 Struve and A. Pogo (**9**), from a study of several hundred orbits, reached the conclusion that the asymmetry of distribution of ω is too pronounced to be a chance occurrence. In R. G. Aitken's tabulation (**10**) of ω one can see that the number of systems for which ω is in the first quadrant exceeds by 50 per cent the number in any of the other three quadrants. The excess is most marked for those orbits whose eccentricities are not greater than 0·10. For these systems of small eccentricity the number in the first quadrant is almost twice as large as the number in any other quadrant. Recently Struve (**11**) once more studied this problem based on his own compilation of the available material. He excluded all cepheid variables, β Cephei or β Canis Majoris stars and other intrinsic variables which are known not to be real binaries. His result shows clearly that there is a marked maximum near $\omega = 15^\circ$ and a broad minimum near $\omega = 225^\circ$. Although the detailed character of the distribution function is new, the essential feature, i.e. the concentration of binary systems with ω in the first quadrant, remains unchanged.

Since Struve's investigation of the distribution of ω, several other studies have been published. V. M. Blanco and A. D. Williams (**12**) have made a similar study based on J. H. Moore's and F. J. Neubauer's fifth catalogue of the orbital elements of spectroscopic binary stars and derived virtually the same result. E. L. Scott (**13**), having made a detailed statistical computation, concludes that the results of her analysis are consistent with Struve's hypothesis that the catalogue values of ω are distorted by systematic errors due to some phenomenon occurring in the vicinity of the binary. Struve's hypothesis mentioned in Miss Scott's conclusion will be the main theme of the following section.

Miss Scott also found that the non-uniformity of the distribution of ω is most pronounced for eclipsing pairs with periods between 2 and 5 days and that among all binary stars the effect is most pronounced for those earlier than spectral type A5. It is very interesting to note that according to M. P. Savedoff (**14**) the same group of stars also shows the largest difference between the spectroscopic and photometric measures of $e\cos\omega$. Savedoff's result therefore further confirms the suspicion that the true distribution of ω may be uniform as the photometric measures indicate, and that the value of ω derived from the spectroscopic measures does not always give the true longitude of periastron because of a distortion of the radial velocity-curve.

4. Distortion of the Radial Velocity-Curve

In the previous section we have given some evidence that the radial velocity-curves of some binary stars may be distorted, although the underlying phenomenon which causes the distortion cannot be understood from the statistical results. A more physical approach is possible when we study the individual binary stars in which the spectroscopic and photometric orbits do not agree. For this purpose we choose U Cephei (B + G) as an illustration.

The star, whose light we observe at all phases except for a few hours during the total eclipse, is of spectral type B8 or B9. The lines are diffuse and broad because of the rapid rotation of this component around its axis. E. F. Carpenter (**15**) first determined the orbital elements from the velocity-curve, but his values ($e=0{\cdot}47$, $\omega=25^{\circ}$) seriously contradict those derived from photometric measures ($e\cos\omega=0$) (**16**). Struve (**17**) and later his associate, R. H. Hardie (**18**), further studied this star spectroscopically. The results of Struve's investigation of this problem are scattered over his three papers (**19**) from which the material of the present section is mostly derived.

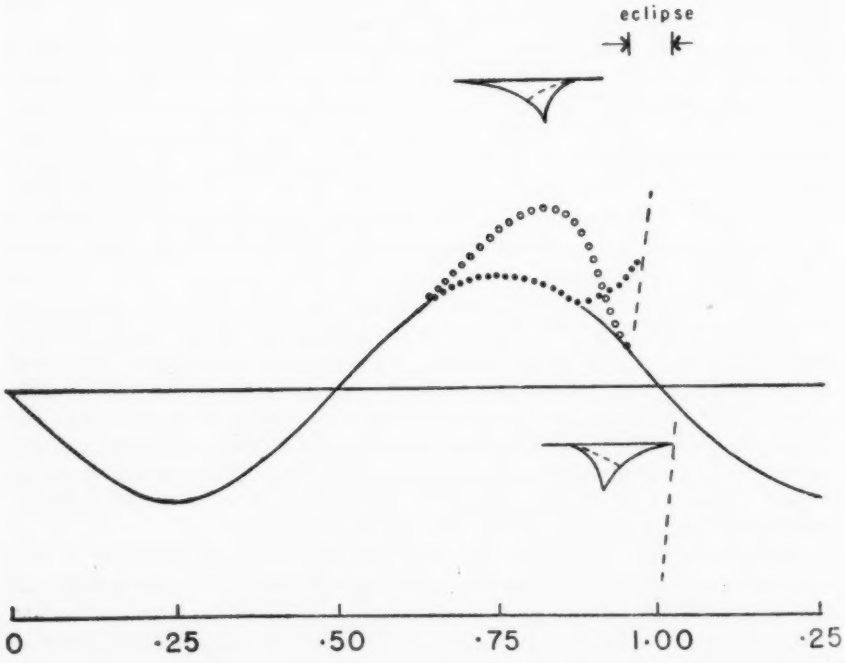


FIG. 2.—*A schematic diagram demonstrating the distortion of velocity-curves of some close binary stars. The solid line represents the true velocity-curve. The excess velocity (denoted by the circled curve) before the principal eclipse is the result of blending of stellar lines with the lines formed in the gaseous stream. The dotted line represents the velocity-curve derived from the centre of the hydrogen wings. Two hydrogen profiles given in the figure illustrate the reversal of asymmetry at a certain phase before the principal eclipse.*

The velocity-curves obtained by Struve and by Hardie show an asymmetry and resemble that measured by Carpenter. Since some close binaries (β Lyrae, etc.) are known to be enveloped in streams of gas which revolve like whirlpools around and inside the system, Struve suggested that the velocity-curve of U Cephei is really symmetrical, as indicated by the photometric measures. Also, he argued that the velocity-curve must give us a radial velocity of approximately zero at the time of principal and secondary eclipses. If we correct the observed curve in this manner we find that there is a large excess of motion away from

the Sun when the principal eclipse has not yet begun. This must be due to a mass of gas which is seen projected upon the apparent disk of U Cephei and whose motion as seen from the Earth is directed away relative to the motion of the star itself. This idea is shown in Fig. 2 (**19**) where a schematic diagram of the velocity-curves is given. The solid line represents the true velocity-curve, the circled line represents the observed curve derived from the cores of H lines and from other lines, and the dotted line represents that derived from the centre of H wings. According to Struve, a stream of gas moving in the same direction as, but generally faster than, the binary component itself, produces absorption lines similar in character and intensity to the absorption lines of U Cephei itself, blending with them, and producing Doppler shifts in excess of those that are normally to be expected. An indication that such blending effects actually take place can be seen by the asymmetry of the profiles of the absorption lines during the critical phases. Such an asymmetry produces the spurious excess in Doppler shifts.

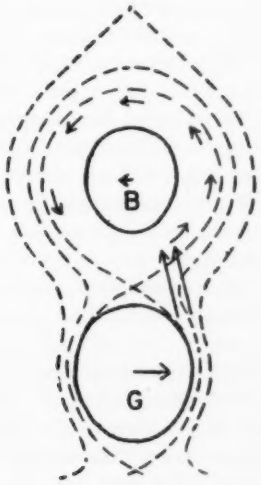


FIG. 3.—*A model of a close binary system of which one component is unstable. The unstable component ejects matter in the direction of the two parallel arrows in the figure. The ejected matter revolves, and at the same time disperses into space, around the hotter star, giving rise to a ring-like structure which can be detected by its emission features.*

A model that Struve proposed some years ago for U Sagittae is reproduced here in Fig. 3, which explains equally well U Cephei, or other similar systems. The large secondary G component has already filled up the inner contact surface and is therefore ejecting mass towards the B component in the manner shown by the two parallel arrows. The distortion of the velocity-curve is due to this stream of ejected gas and is therefore most pronounced at phases before the principal eclipse. The mass ejected by the G component will disperse around the B component as an extended nebulosity, giving rise to the emission feature which will be discussed in the next section. A part of the nebular gas will fall into the B component and the rest escapes from the system. At present we have

no knowledge about the relative importance of the two ways of dissipation of the ejected mass.

The process of disentangling the blended absorption lines produced in part by the atmosphere of the star itself and in part by the gaseous streams is often a difficult task. In the case of the H lines we can well believe that the wide wings which are the evidence of the Stark effect must be formed in the stellar atmosphere where the density is great compared with the stream of gas surrounding it. An H absorption line formed in the stream must be comparatively sharp because of low pressure. This line would therefore appear as a deep core superimposed on the broad wing of stellar origin. Because of the relative motion of the gaseous stream with respect to the star, the core will not fall exactly on the centre of the stellar line, resulting in an asymmetry of the observed profile. Indeed the asymmetry of the H line at different phases is quite conspicuous as can be seen in Fig. 2 where two representative profiles are given. We can determine the centre of the broad wings and thereby derive the motion of the star. This being done, we find that while the radial velocity thus determined does conform to the symmetrical sine curve adopted to represent circular motion for a large section of the curve, there is a serious discrepancy immediately before the eclipse where the velocity corresponding to the centre of the broad wings increasingly deviates, in the positive direction, from the sine curve until the H line disappears in the eclipse. When the line reappears after the eclipse, the velocity corresponding to the centre of the broad wings deviates, in the negative direction, from the sine curve, but gradually merges into the latter. This effect looks like the well-known Rossiter rotational disturbance (**20**) (shown as the broken line in Fig. 2), but it cannot be explained by it because the phenomenon is present even before any part of the B type star is covered by the G type star. Moreover, this remarkable result is not shown by other lines, such as helium, magnesium, silicon, etc. It must be caused by a phenomenon that is unrelated to the stream of gas.

Struve proposed an explanation of this strange phenomenon based on the influence of rotation on the Stark effect. If the B star is tidally distorted to form approximately a cigar-shaped body, then at the tips the Stark effect is weak. Hence, the spectral lines contributed by the bulges will be narrower than those produced by the central region. Consequently the asymmetry of the observed H line will depend upon the orientation of the star. A quantitative investigation of this idea has been carried out by Hardie (**18**). His result indicates that the reversal of line asymmetry during the partial phases can be interpreted in terms of this hypothesis.

Hardie (**18**) has corrected the velocity-curve by the displacement of the centre of the broad H wings from the core. The velocity-curves reduced in this manner at two epochs (1943 and 1949–1950) both yield small eccentricities, but they do not agree with each other. This is a disturbing phenomenon which has not been explained.

The distortion of the velocity-curve is not limited to U Cephei. Indeed, the radial velocity observations of many eclipsing binaries can be explained in a similar way. Among them are RZ Scuti (**21**), RY Persei (**22**), RW Tauri (**23**), SX Cassiopeiae (**24**), RX Cassiopeiae (**25**), U Coronae Borealis (**26**), TT Hydrae (**27**), RS Sagittarii (**28**), R Arae (**28**), U Sagittae (**29**), etc. From the published

data we can conclude that nearly all eclipsing variables whose periods are approximately between 2 days and 5 days, and in several other systems of longer periods, gaseous streams produce blended absorption lines which distort the velocity-curve primarily at phases between $0{\cdot}8P$ and $1{\cdot}0P$ and to a somewhat lesser extent between $1{\cdot}0P$ and $0{\cdot}2P$. This conclusion explains also the anomalous distribution of ω of eclipsing binaries with periods between 2 days and 5 days (**13**).

5. Problems of Emission Lines

The existence of gaseous streams enveloping binary systems postulated in the previous sections can be proved more directly by the emission lines which occur in many eclipsing binaries, as was first noticed by A. B. Wyse (**30**). When the streams are seen projected upon the disk of the bright component of the binary, they produce absorption lines which blend with the stellar absorption lines and distort the velocity-curve. But when the streams are not seen projected upon the disk of either star, they are observed as emission lines if there are enough emitting atoms in the stream. The visibility of emission lines depends upon the total radiative flux received by us from the gaseous streams as compared with that from the star. Because of the background of strong stellar radiation, the emission lines are sometimes invisible. When the light of the system is cut down during the eclipse, the emission lines stand out.

As an illustration let us consider RW Tauri, the emission feature of which was first observed by Wyse (**30**). A. H. Joy (**31**) found that the bright lines consist of two components. At second contact the red emission component appears alone, and at third contact the violet component alone is visible, while at mid-eclipse no emission is seen. Joy suggested that these emission lines originate in an extended gaseous ring which surrounds the equatorial region of the primary B9 star in a manner not unlike the gaseous ring proposed by Struve for Be stars (**32**). The larger K0 star first occults the approaching side of the ring, leaving only the red emission component. Then the entire ring is eclipsed and no bright lines are visible. Finally, the receding side of the ring is occulted and we see only the violet emission component. Since the rotating ring can be regarded as a special case of a gaseous stream, we shall retain in our discussion the term "gaseous stream" for generality.

The problem of emission in binary systems was more extensively studied by Struve and his associates. The most important results of their investigations have been summarized in the three papers mentioned before (**19**) as well as in other articles (**1, 33, 34**). The following discussion is mostly based on these papers.

Among 30 binaries which show total or near total eclipses and which have been studied with sufficient care during the eclipse, about 16 have been found to possess bright lines of hydrogen and sometimes of other elements, which vary in intensity in precisely the same manner as do the bright lines of RW Tauri. All these stars are listed in Table I. The classifications are taken from W. P. Bidelman's catalogue (**35**).

The observations of these stars are difficult, because most of the variables investigated have photographic magnitudes between 10 and 13 at principal minimum. Moreover, the eclipses last only a few hours and the changes in the

TABLE 1

Eclipsing Binaries with Emission Rings

Star	Magnitude	Period (days)	Sp. Type	Reference
SX Cas	9·5–10·7	36·6	cA6e+gG6	24
RX Cas	8·8– 9·5	32·3	gG3+gA5e	25
RW Tau	8·0–12·3	2·8	gB9e+sgK0	23, 31
RW Per	9·9–12·0	13·2	A5e+gG0	1
RS Cep	10·2–11·9	12·4	A5e+G	1
DN Ori	9·8–10·9	13·0	A2e+gF5	1
RY Gem	8·5–11·3	9·3	A2e+K2	38
UX Mon	8·7– 9·7	5·9	A7e+G2	40
S Vel	8·0–10·5	5·9	dA5e+gK5	39
TT Hya	7·5– 9·3	7·0	A3e+G5	27
RZ Oph	10·0–10·9	262	F3eIb+cK5	22
U Sge	6·4– 9·0	3·4	B9e+gG2	29
SW Cyg	9·3–11·9	4·6	A2e+K0	1
VW Cyg	9·7–12·4	8·4	A3e+G5	1
W Del	9·4–12·8	4·8	A0e+G5	1
AQ Peg	10·0–12·7	5·5	A2e+G5	1

bright lines caused by the occultation of the gaseous rings by the large late-type component are often very rapid.

The frequency distribution of the periods of all known eclipsing stars has a steep maximum between one and two days. But of the 16 stars which show ring-like emission, the periods, ranging from 2·77 days for RW Tauri (**31**) to 261·9 days for RZ Ophiuchi (**22**), are all greater than the period corresponding to the most frequent occurrence. Therefore the emission features exist preferably in the eclipsing binaries of longer periods. A part of this preference may be due to observational selection, because it is difficult to observe the emission lines when the eclipse is of very short duration.

It is interesting to note that in all systems the star of earlier spectral type is the one eclipsed when the emission lines show the characteristic variations in intensity. This is true even when the component of earlier spectral type is not the more luminous one (in the photographic region). An example can be found in the system RX Cassiopeiae (**25**). Struve further suggests that there exists a certain relation between the period and the spectral type of the earlier component of the system, i.e. the shorter the period, the earlier is the spectral type.

The rings show a velocity of rotation of the order of several hundred km/sec, and in all cases the direction of motion is the same as that of the binary system. A relation exists between the velocity V_{em} of rotation of the ring as derived from the shift of the emission lines and the period P of the binary:

$$V_{em}{}^3 \propto P^{-1}. \qquad (5)$$

If the masses of the systems were all the same, this would be the form of Kepler's third law. But we must remember that the masses of the stars are not all the same and that the periods are those of the binary components but not those of the rings. Nevertheless, this is an interesting relation which tells us that for the longer periods the space available for the rings is larger than for the shorter periods. Thus, it is not surprising that the eclipses of the rings tend to be total for systems of short periods and annular for long periods. From the

dilution effect (**36**) we find that the extent of the gaseous ring is not larger than two or three times the diameter of the brighter component. Also, the total emission from the rings is a function of the period. In the long-period binaries the bright lines are visible even in full light, while in the short-period binaries the bright lines can be seen only during very deep total eclipses. Bright H lines are not observed in U Cephei possibly because the system is too small.

If the systems are selected at random, without regard for period and spectral type, and with the sole restriction that the eclipse be total or nearly total, the fraction of systems which show double emission lines is about one out of four or five. But the number of binaries which have gaseous streams around the systems must be much greater. Because of small size or of unsuitable level of ionization, a gaseous stream becomes unobservable in many systems. Consequently the existence of hydrodynamic motion of gases may be a common, or even a universal, phenomenon in close binary systems.

A gaseous ring rotating around one of the components of a close binary system is most likely unstable. For this reason, the term "gaseous ring" should be used in a broad sense, i.e. the ring may not completely detach itself from the star.

According to the model given by Fig. 3 the gaseous streams surrounding the B component come from the ejected mass by the G component. Most likely the ejection takes place in the *xy* plane. From symmetry considerations we expect that the *z*-component of the ejected velocity vanishes. Accordingly, although the exact orbit of the gas after ejection is difficult to ascertain theoretically, it must be confined to the equatorial plane. According to Section 2, the zero-velocity surface is determined by the initial conditions. In the present case the initial condition is the ejection velocity at the point of ejection on the surface of the G component. Hence the degree of violence of ejection (or prominences) of the G component determines the extent to which the ejected matter can spread out. Ionization by radiation from the B component takes place most effectively after the ejected gases have dispersed into a tenuous nebulosity; consequently we do not observe any appreciable systematic difference in the strength of emission lines before and after the principal eclipse, while the absorption lines formed in the gaseous stream show up most clearly before the principal eclipse. Since the ejection, like the prominences, may not be a steady process, we cannot expect the emission features in the eclipsing variables to be invariant. Indeed, Wyse (**30**) found that the bright lines observed in RW Tauri during principal eclipse not only vary in intensity but sometimes disappear completely. Further study of the same star by Joy (**31**) confirms Wyse's conclusion. Emission lines were absent at 5 out of 20 minima during which Joy has made spectroscopic observations. A wide variation in intensity of the bright lines exists on the plates taken during the remaining 15 minima. Also, D. H. McNamara (**29**) has taken some plates of U Sagittae which show clearly emission lines, whereas plates taken previously show no emission features at all (**19, 37**). Variations of emission lines in other eclipsing variables might have escaped our attention. The formation or dissipation of the gaseous rings in such binary systems must be an exceedingly rapid process. When the ejection process on the surface of the G component stops, the gaseous stream disappears because of its dynamical instability. As soon as the G component ejects a certain amount of matter, a gaseous ring is formed around the B component. In other

words, we believe that the existence of emission lines depends on the continuous supply of mass by the G component.

Finally, let us compare the gaseous rings observed in binary systems with those in Be stars. The character of the double H emission lines in the two cases are similar, hence the general structures of the gaseous rings cannot be too different. On the other hand, there is a great deal of difference between the two groups of stars. In the Be stars the overwhelming majority occurs in the earlier subdivisions of class B and in the O stars. In the binaries the phenomenon is associated mostly with the late B and early A stars. Indeed we can see that the earlier component of every binary in Table I belongs to the spectral range B9–A7, with the exception of RZ Ophiuchi which has a supergiant as its principal component. In spite of the difference in the temperatures of the exciting stars, the ionization of the gaseous rings of the two groups of stars is approximately the same. Struve (**33**) has roughly calculated from the equation of ionization that with proper estimation of the dilution factors the electron pressure in the rings of binaries is of the order 10^{-2}–10^{-3} times that in the rings of Be stars. The extremely low pressure existing in the gaseous ring in the binary system is consistent with the observed result that the H lines show no broadening of the kind produced by the Stark effect.

6. Beta Lyrae

We shall consider β Lyrae because it is, astrophysically, one of the most interesting close binaries in the sky. Although its spectrum has been investigated more minutely than that of any other star, the physical nature of this remarkable object has not yet been fully explained. The spectrum consists of four distinct patterns of spectral lines which can be taken as coming from several distinct sources (**41**):

(1) A normal giant-type absorption spectrum of class B9, which shows a regular variation in velocity with a range of 367 km/sec and with a period of approximately 13 days. This source is identified with that component of the binary system which is obscured at the primary eclipse.

(2) An abnormal absorption spectrum resembling spectral type B5, varying in intensity, but showing little systematic variation in velocity during the entire cycle, although the lines show rapid irregular changes in radial velocity between successive cycles and also during a single cycle when the exposures are separated by 1 or 2 hours (**42**).

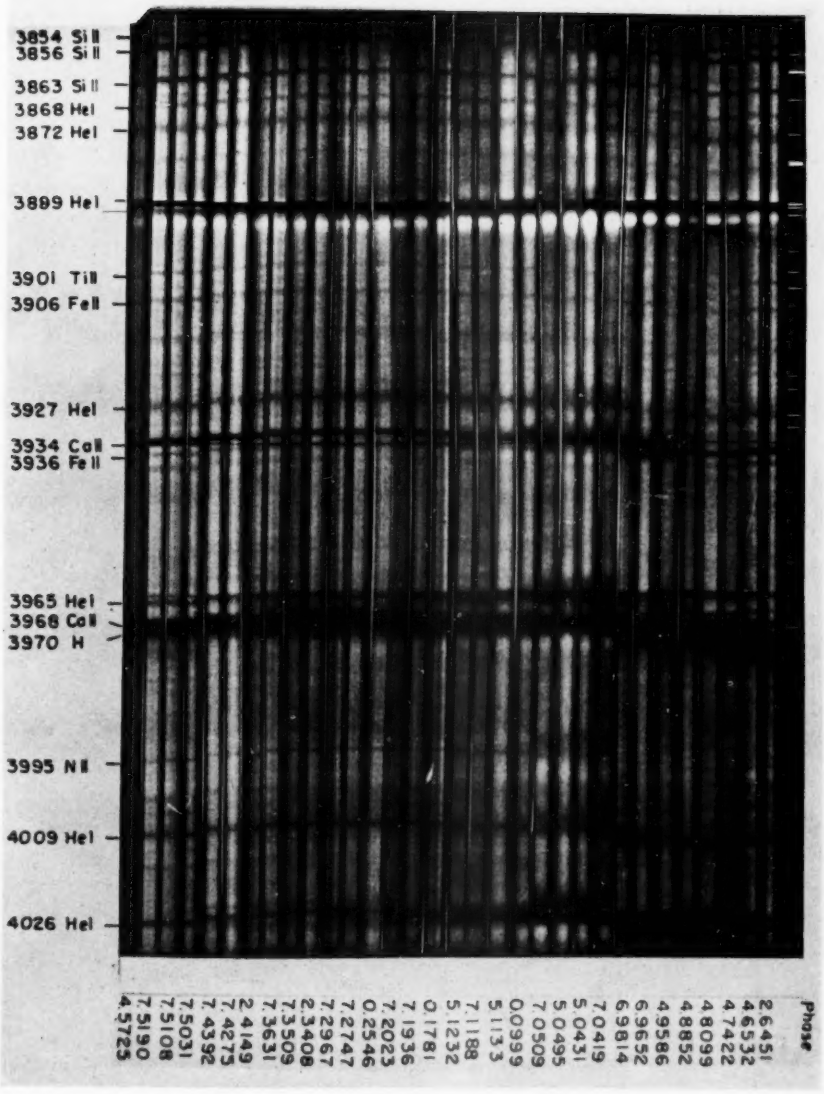
(3) An emission spectrum of variable intensity consisting of broad emission bands. The emission spectrum is associated with the B5 absorption lines and comes from the same source.

(4) Besides the three permanent features the spectrum shows two sets of additional lines which appear only in a small fraction of one cycle respectively before and after the mid-eclipse.

The B5 absorption spectrum does not come from a star, but originates in an expanding nebulous mass of gas which surrounds the binary system (**43**).

Plates 17, 18, 19.

Spectra of β Lyrae in 1955 by O. Struve and J. Sahade with the coudé spectrograph of the Mount Wilson 100-inch telescope. Phase 0·0000 corresponds to the middle of the principal eclipse. The number in front of the decimal point designates the number of the 12·9 days cycle, counted from an arbitrary beginning.



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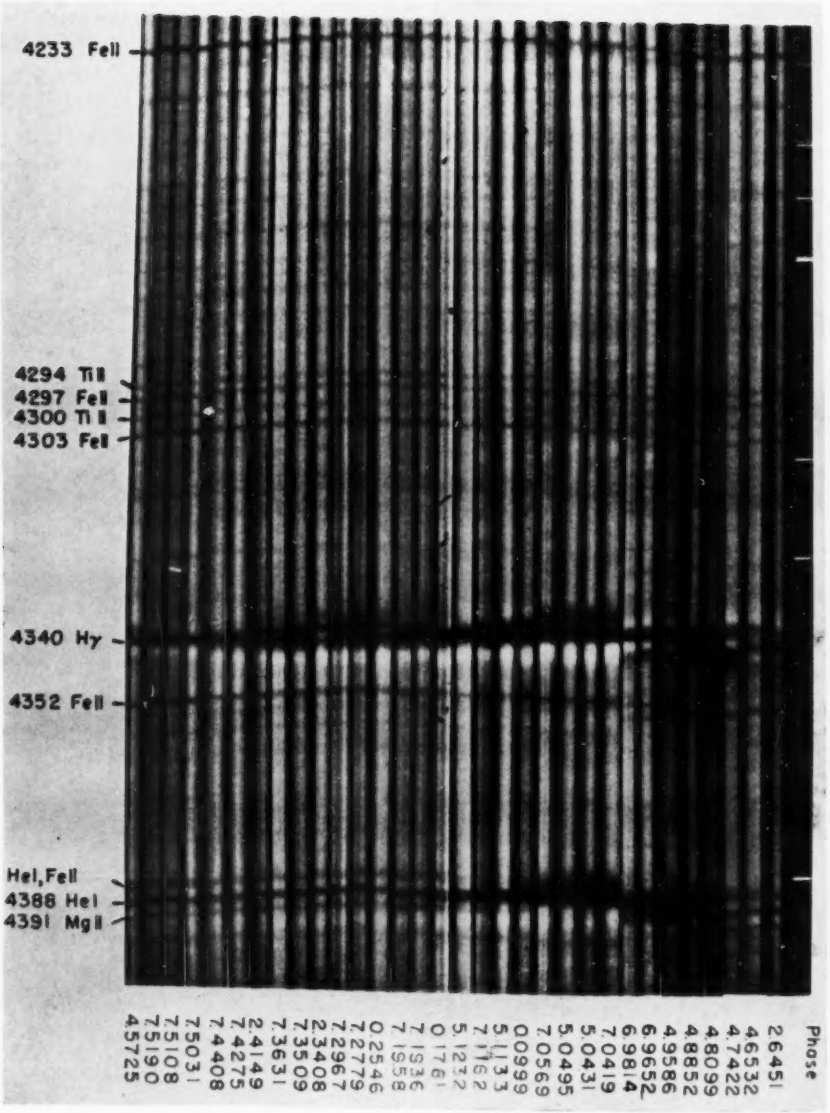
4233 FeII
4294 TiII
4297 FeII
4300 TiII
4303 FeII
4340 Hγ
4352 FeII
HeI,FeII
4388 HeI
4391 MgII
Phase
.26451
.46532
.47422
.48099
.48852
.49586
.69652
.69814
.70419
.50431
.50495
.70569
.00999
.51133
.71162
.51272
.01761
.71536
.71558
.02546
.72779
.72967
.23408
.73509
.73631
.24149
.74275
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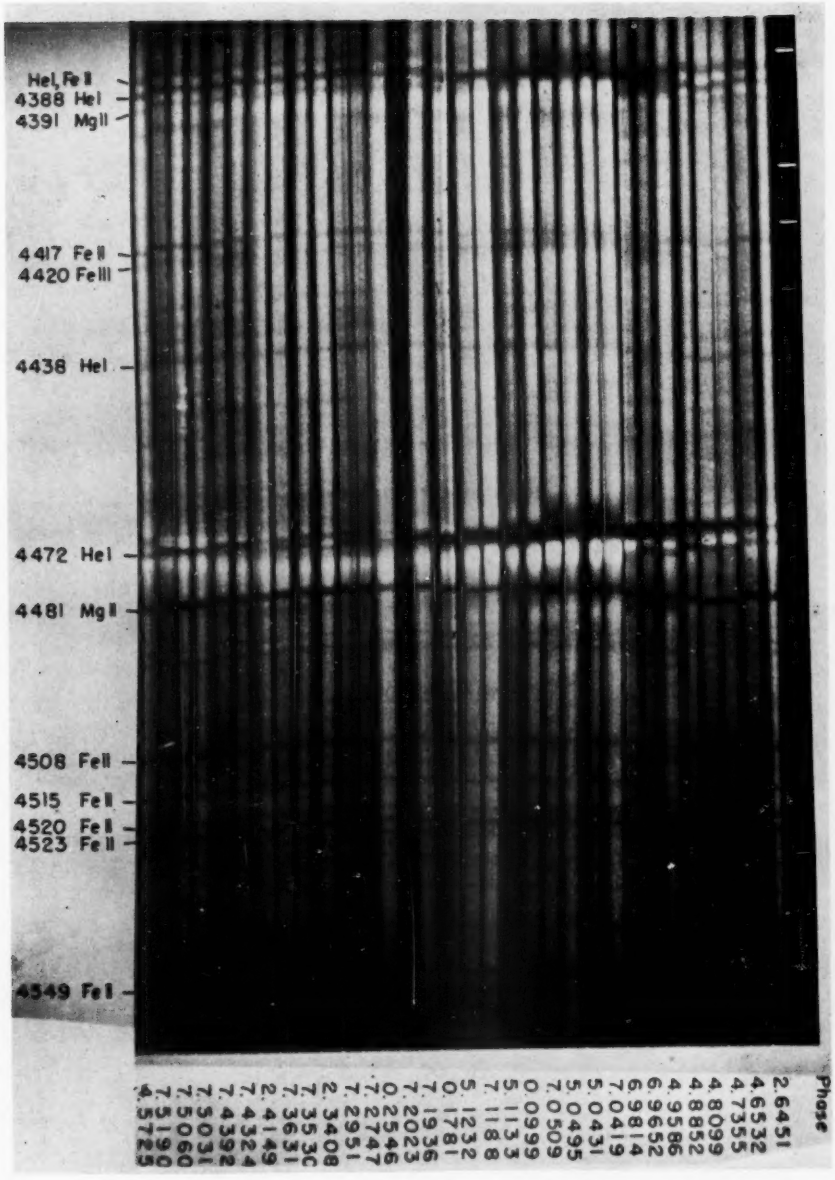
.75108
.75190
.45725


HeI, FeII
4388 HeI
4391 MgII
4417 FeII
4420 FeIII
4438 HeI
4472 HeI
4481 MgII
4508 FeII
4515 FeII
4520 FeII
4523 FeII
4549 FeII
Phase
2.6451
4.6532
4.7355
4.8099
4.8852
4.9586
6.9652
6.9814
7.0419
5.0431
5.0495
7.0509
0.0999
5.1133
7.1188
5.1232
0.1781
7.1936
7.2023
0.2546
7.2747
7.2951
2.3408
7.3530
7.3631
2.4149
2.4324
7.4392
7.5031
7.5060
7.5190
4.5725

The faint component is not observable spectroscopically even at the time of primary minimum, but the photometric observations indicate that it is of spectral class F and is smaller and less massive than the B9 component.

The B5 spectrum undergoes large variations in intensity. It has very strong lines of H and He I. In any normal stellar spectrum having lines of helium similar to those of the B5 spectrum in β Lyrae, we always expect to see strong lines of Mg II 4481, Si II 4128 and Si II 4131. But in the B5 spectrum of β Lyrae these lines are quite weak. Since these lines are very sensitive to the effect of dilution of radiation, it immediately follows that the radiation which forms the B5 spectrum comes from a nebula at a great distance from the exciting star (**36**). Struve has actually estimated from the dilution factor the distance between the nebulous gas and the centre of the B9 component to be three times the radius of the latter (**41**). Thus, the dimensions of the nebula are comparable to those of the binary system and the secondary (invisible) component is enveloped in the nebular gas. From the radial velocity measurements at different phases, the observable layers of the nebular gas show, in general, a state of expansion as well as rotation.

Miss J. R. Gill (**44**) has studied the changes in equivalent widths of some B5 lines during the cycle. They all vary in the same manner and clearly represent varying amounts of nebular material projected upon the disk of the B9 component. The change in line intensity repeats itself roughly, but not exactly, in every cycle. However, no change in ionization has been detected either during a cycle or from cycle to cycle.

Just before the principal mid-eclipse the spectrum shows additional absorption lines which are strongly displaced towards the red side of the spectrum and which are not seen at any other part of the cycle. Immediately after the mid-eclipse we see another set of absorption lines with a strong displacement towards the violet. They remain visible during only a fraction of the period and disappear for the rest of the cycle. These highly displaced lines are frequently referred to as the "satellite lines". Struve's interpretation of the satellite lines is that (**41**) they come from two streams of gas—one receding as we observe from a point just preceding principal mid-eclipse, and the other rapidly approaching, as we observe from a point immediately after principal mid-eclipse. Before mid-eclipse the F-type star obscures the approaching stream. After mid-eclipse, it obscures the receding one. The two streams must therefore be located somewhere in the space between the two components.

The spectral characteristics of these two sets of satellite lines are quite different from each other (**45**). The approaching stream is strong. Its lines are broad, ranging in velocity between -80 and -360 km/sec, and remain roughly unchanged between phases 0·04 and 0·05. But at phase 0·09 and until phase 0·12 they are replaced by fairly strong emission components. Evidently this stream remains projected against the B9 disk until phase 0·09, after which it is seen partly against the space between the two stars, and partly against the disk of the invisible F component. This stream is strong in H and He I, but it also shows lines of N II, Ca II and Fe III (4419·6). There is little or no dilution effect: the satellites of He I 3888 and 3965 are not enhanced as are the corresponding B5 lines. No doubt this stream is coming from the B9 star and is seen fairly close to the surface of the B9 component, but its ionization is considerably greater, probably because of reduced density and increased temperature.

The receding stream observed just before mid-eclipse is characterized by much lower ionization and lower excitation (corresponding to approximately type A2) and is probably coming from the cooler component. The lines are less broadened by the Doppler motion than the satellites immediately after the mid-eclipse. Their edges correspond, in the mean, to velocities of 140 to 200 km/sec, but these velocities increase as we pass from phase 0·95 to phase 0·98, approximately, and in all probability the range also increases. There is

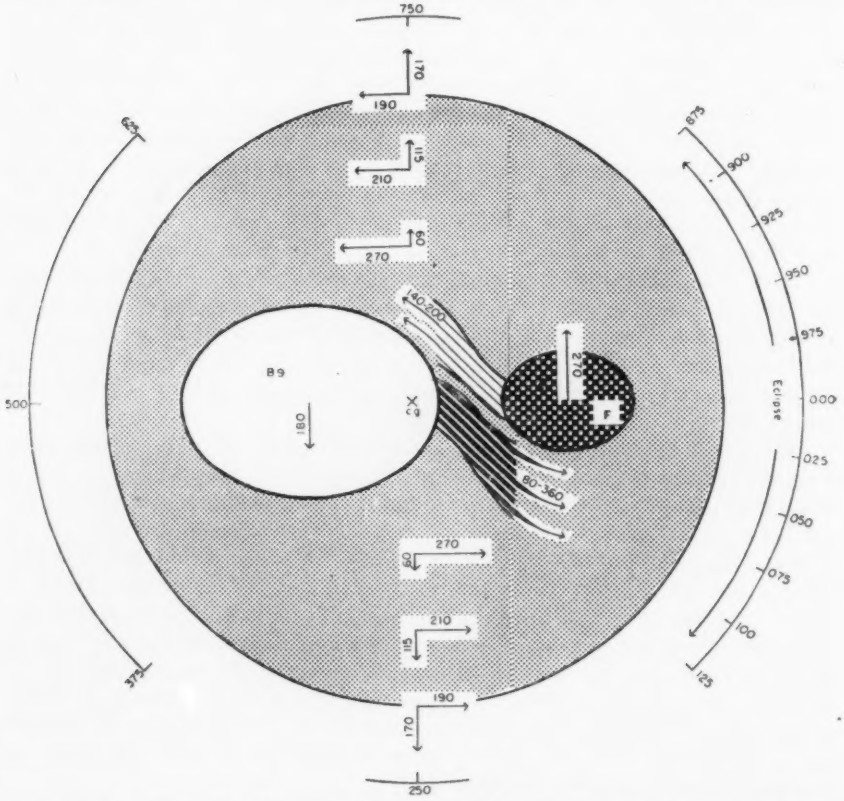


FIG. 4.—*A model of β Lyrae. Two observable streams, one ejected by each component, lead to the hypothesis that both components in the system are unstable. The nebulous gas surrounding the system shows both rotation and expansion. The numerical values attached to the arrow-heads at different places denote the observed velocities in km/sec at those places.*

some indication that immediately after the disappearance of the red absorption satellites they are replaced by weak emission components. This is best seen in Si II, Mg II and perhaps Ca II (**45**).

Following these observational results Struve (**45**) has sketched a physical model for β Lyrae in his recent paper given at the I.A.U. meeting at Dublin. The model, shown in Fig. 4, is an improved version of the same star proposed more than ten years ago (**41**). The numerical values associated with the arrow in the figure represent velocities in km/sec (with respect to the centre-of-mass of the system) observed at the spot. Such a picture immediately suggests that

streams are produced by the instability of both component stars, near the inner Lagrange point, which G. P. Kuiper (**5**) has designated as instability of class A. This agrees with the theoretical results by V. A. Krat (**46**), A. N. Dadaev (**47**), and A. Kranjc (**48**). It also permits us to connect the phenomena in β Lyrae with those considered in the previous sections.

Since both components approach the limit imposed by the inner contact surface, the ejected mass will probably escape from the system. Indeed, the observation shows that the nebulous gas surrounding the system expands with a considerable velocity. Eventually the mass in the expanding layers will escape into outer space.

It should be recognized that the escape of gases to interstellar space cannot be directly explained by the theory of zero-velocity surfaces in binary systems. At great distances from the two stellar components (or for large values of x^2+y^2 in equation (3)) the equipotential surfaces resemble those of a single star whose mass is unity. Why, then, do we observe an outer expanding atmosphere in β Lyrae? The theory only explains the formation of a stream trapped between appropriate zero-velocity surfaces. Because the velocities of the streams are smaller than the escape velocity from the system, the escape to infinity in a close binary should not differ appreciably from a similar escape in the case of a single star. There is, however, ample evidence that if an outer shell has somehow been formed—in a binary or a single star—this shell is often, if not always, accelerated outward, perhaps by radiation pressure. We know that this happens in single supergiants like α Orionis and in wide visual binaries like α Herculis. In the case of β Lyrae the formation of a stream belonging to the system is aided by the binary nature of this star. The acceleration outward then proceeds as it would in a single star having the combined mass of the pair.

It can be shown that when the total mass of a system decreases, its period increases (**4**, **49**). Actually, the increase of periods of β Lyrae has been a well-established result of centuries of observations. Z. Kopal (**50**) has given a value for the rate of increase equal to 9·38 seconds per year. Using this value and his determination of the mass of the principal component ($52{\cdot}2\odot$) we find (**4**) that the rate of loss of mass of the star is $d\mathfrak{M}/dt = -2{\cdot}82\times10^{22}$ gm/sec. Kopal's value of the rate of increase in period is apparently an average value over the time since the first reliable observation. If we can represent the time of eclipse by a quadratic expression of the number of cycles, the instantaneous rate of the increase of period per year should be twice that of the average value. Consequently we should have at the present time

$$\frac{d\mathfrak{M}}{dt} = -5{\cdot}64\times10^{22}\ \text{gm/sec.} \tag{6}$$

Following J. A. Crawford and R. P. Kraft (**51**) we can estimate the rate at which mass is emitted by the star in terms of its luminosity and find

$$\frac{d\mathfrak{M}}{dt} = -7{\cdot}2\times10^{22}\ \text{gm/sec.} \tag{7}$$

The agreement between the two values of $d\mathfrak{M}/dt$ obtained from two independent methods should be regarded as satisfactory. This strongly supports the idea that the massive component (or both components) of β Lyrae is in the stage of evolutionary expansion and is rapidly losing its mass. If we can trust the third-order term in the empirical formula of R. Prager (**52**) for the time of

eclipse in terms of the number of cycles, the rate of losing mass is gradually slowing down.

The lines of the B9 component which are not appreciably complicated by emission or by the B5-type absorption remain almost constant in intensity, except during the primary eclipse. The fainter lines become broad and hazy at certain phases during the principal eclipse. This cannot be due to Rossiter's rotational disturbance because the equatorial rotational velocity of the B9 component is only 40 km/sec as determined from the profiles observed outside of eclipse (**53**). The broadening at mid-eclipse is not the same in different cycles, but it is always more pronounced after mid-eclipse than before, and it seems to produce lines which are about twice as broad as is consistent with the rotation of the B9 star. This is almost certainly an effect of electron scattering in the highly ionized stream of gas which flows from the B9 star toward the observer. The temperature and electron density are of the right order of magnitude to produce the observed broadening (**45**). This broadening does not repeat itself exactly in different cycles. It is also present outside of eclipse, but is then much less conspicuous. Apparently, the effect of electron scattering contributes relatively more to the total brightness when a part of the B9 star is obscured by the invisible F star. Most of the electron scattering is probably produced in the accumulation of gas in the stream which originates near the point L_1 from the B9 star. This mass is mostly eclipsed by the B9 and F components during the second half of the cycle. Hence, between phases 0·5 and 1·0 the effect of electron scattering is less obvious than between phases 0·0 and 0·5. Further investigation of this point is being carried out in Berkeley.

Some important problems connected with β Lyrae remain unsolved. The photometric measures indicate that the B9 component must be large. Z. Kopal (**50**) found that $a_1 + a_2 = 8 \times 10^6$ km. As a rough approximation we can set the radius of the B9 star equal to $(a_1 + a_2)/2 = 4 \times 10^6$ km. By assuming synchronization of the orbital motion and the rotation of the star, we obtain an equatorial velocity of 230 km/sec (**54**) as compared with the observed value of 40 km/sec. It seems, therefore, that the radius determined from the photometric study is too large to be consistent with the observed rotational velocity. Some spectroscopic criteria, such as the number of H absorption lines and their Stark-effect broadening (**55**) also suggest a smaller radius. On the other hand, from the mass function inferred from the radial velocity-curve of the primary (B9) component, the luminosity of the B9 component must correspond to that of a supergiant, if we apply the mass–luminosity relation to the B9 star by assuming a reasonable mass ratio. This agrees with the photometric measures. This question cannot be settled now and β Lyrae remains for years to come one of the most interesting objects in the sky to be investigated.

7. The Failure of the Mass–Luminosity Relation and its Significance

The magnitude difference, or the relative brightnesses, of the components of an eclipsing binary can be determined from the photometric observations, and, if it happens to be also a double-lined system, the magnitude difference can be further checked by spectroscopic means as was done by R. M. Petrie (**56**). On the other hand, the masses of the components can be estimated from the mass function which is derivable from the velocity-curve. We should expect that the

observed magnitude and mass differences of the two components of a binary satisfy the mass–luminosity relation. Actually, there are many binary systems for which this is not true. This anomalous phenomenon indicates a failure of the mass–luminosity relation. Among these systems, the W Ursae Majoris binaries and the Algol-type variables form two of the most conspicuous groups (**57**). The spectra of both components of a W Ursae Majoris system are observable. Moreover, the spectral types of the components are nearly always alike and their luminosities differ only slightly. Accordingly, the mass-ratio should be nearly equal to unity. But the velocity-curves of different systems yield values of the mass ratio scattered around 2. Therefore, at least one of the components must violate the mass–luminosity relation. The departure from the mass–luminosity relation of the W Ursae Majoris systems may be due to the close proximity of the two components so that a common envelope exists, in which matter and energy of the two components exchange freely. The exchange of energy and matter in the envelope suppresses any difference—both spectroscopic and photometric—of the two components that they may have underneath their common envelope.

But the most interesting problem associated with the failure of the mass–luminosity relation comes from the fainter components of some Algol-type variables (**3**). The radial-velocity amplitudes of the brighter components of such variables are very small and consequently indicate small values of the mass function. A quantitative analysis can be given as follows.

Let K_1 be the semi-amplitude of the velocity-curve of the primary component, which alone is observable. We can write the mass function $f(\mathfrak{M})$ as

$$f(\mathfrak{M}) = \frac{\mathfrak{M}_2^3 \sin^3 i}{(\mathfrak{M}_1 + \mathfrak{M}_2)^2} = cK_1^3 P(1-e^2)^{3/2}, \tag{8}$$

where c is a numerical constant depending only upon the choice of unit. Now we are concerned with systems of relatively short period, so that $1-e^2 \simeq 1$. Introducing the mass ratio,

$$\alpha = \mathfrak{M}_1/\mathfrak{M}_2 \quad (\geqslant 1), \tag{9}$$

we can re-write equation (8) as

$$K_1 = \left[\frac{\mathfrak{M}_1}{\alpha(1+\alpha)^2 cP}\right]^{1/3} \sin i. \tag{10}$$

Hence for a given pair of values of P and $\mathfrak{M}_1$, K_1 can have any value between the upper limit, K_0, given by

$$K_0 = \left(\frac{\mathfrak{M}_1}{4cP}\right)^{1/3}, \tag{11}$$

and zero. K_0 is determined by setting, in equation (10), α equal to the smallest permissible value and $\sin i$ to the largest permissible value. Struve has plotted the values of K_1 against $\log P$ separately for different spectral types, B, A, F, etc. (**58**), which corresponds approximately to a constant $\mathfrak{M}_1$ for each diagram. For the ordinary spectroscopic binaries, the points in the diagram indeed scatter nearly uniformly below the limiting curve given by equation (11) as expected. The distribution of the points on the $K_1 - \log P$ diagram tends to zero abruptly at a vertical line at about $P = 1{\cdot}2$ day for the B stars and $P = 0{\cdot}8$ day for the A stars. This is obviously due to the limitation imposed by the radii of the stars. We expect that a similar plot for the eclipsing binaries would show a

concentration of points near the upper limiting curve given by equation (11) because of the large inclinations necessarily implied in these systems. Actually, a large fraction of the eclipsing variables fall far below the limiting curve.

We can now make some estimate for the mass of the secondary components of the Algol-type variables. Since they are eclipsing binaries, we set $i=90^\circ$. Eliminating $\mathfrak{M}_1/(cP)$ from equations (10) and (11), we obtain

$$\alpha(1+\alpha)^2=4\left(\frac{K_0}{K_1}\right)^3, \tag{12}$$

the value of K_0 for each P being obtainable from the K_1 *vs* $\log P$ diagram for the ordinary spectroscopic binaries. By eliminating $\mathfrak{M}_1$ from equations (10) and (11), we have automatically assumed that the primary components of the same spectroscopic type have the same mass for both the ordinary spectroscopic binaries and the eclipsing binaries. Of course we are concerned only with the main-sequence components in both kinds of binaries, and there is little doubt about the assumption that the primary component of an Algol-type variable is a normal dwarf.

We have computed the real root of equation (12) for different values of K_0/K_1 and given the result in Table II. For example, J. Sahade (**59**) found that $K_1=11{\cdot}5$ km/sec and $P=3{\cdot}28$ days for XZ Sagittarii—one of the Algol-type variables. From the limiting curve in the $K_1-\log P$ plane for the A components of ordinary spectroscopic binaries we obtain $K_0\simeq 120$ km/sec. Therefore we have $\alpha=16$ according to Table II. Computing directly from equation (8) by setting $i=90^\circ$ and $e=0$, we have $\mathfrak{M}_2=0{\cdot}17\odot$, and $\alpha=17{\cdot}6$ if $\mathfrak{M}_1=3\odot$ is assumed (**59**). Table II, therefore, provides a convenient way for determining the mass ratio without assigning a definite numerical value to the mass of the primary component, because in our calculation we have assumed that the mass of the primary component is equal to the mass of the primary component of ordinary spectroscopic binaries of the same spectral type and the same period.

TABLE II

Computed Relation between Semi-amplitude of Velocity-Curve and Mass Ratio

K_0/K_1	α	K_0/K_1	α	K_0/K_1	α	K_0/K_1	α
1	1·00	5	7·28	9	13·64	13	19·98
2	2·54	6	8·87	10	15·22	14	21·57
3	4·12	7	10·46	11	16·81	15	23·15
4	5·70	8	12·04	12	18·37	16	24·74

A mass ratio equal to 16 would predict according to the mass–luminosity relation a difference in luminosities of the two components

$$\Delta M_{vis}\simeq 11{\cdot}5 \text{ mag},$$

while directly from the photometric solution we have

$$\Delta M_{vis}=2{\cdot}5 \text{ mag}.$$

Hence the companion is 4×10^3 times more luminous than would be consistent with the mass–luminosity relation. Sahade's new observation (**59**) for the same star yields a value of $K_1=23$ km/sec. The discrepancy is therefore considerably reduced, but the observed luminosity is still 10^2 times that which would be expected from the mass–luminosity relation. The departure from

the $\mathfrak{M} - L$ law thus encountered is not limited to the XZ Sagittarii system. As a matter of fact, it illustrates a general tendency of the fainter components of most Algol-type variables. Their velocity-curves all show small amplitudes and as the previous analysis shows, they correspond to large values of α. Actual determination indicates that the masses of the secondary components range from $0{\cdot}2\odot$ to $1{\cdot}0\odot$, but both their luminosities and effective temperatures are much too high to be compatible with such small masses, if the $\mathfrak{M} - L$ relation applies. The velocity-curve could not be disturbed by blending of spectral lines of the two components because of their differences in both spectral type and magnitude. Consequently, the result that the fainter components of the Algol-type variables fail to satisfy the mass–luminosity relation is intrinsic and provides us with one of the most interesting problems in close binary astronomy.

When these fainter components are plotted in the H–R diagram, they fall in the region of subgiants, being one to three magnitudes above the main sequence and between F and K. When these stars are plotted in the $\mathfrak{M}$ *vs* L diagram, they lie above the conventional $\mathfrak{M} - L$ curve as can be expected from the previous discussion. Also, they show in both diagrams a large amount of scatter.

If, however, we plot, following P. P. Parenago and A. G. Massevich (**60**), $\log(\mathfrak{M}/R)$ against $\log(L/\mathfrak{M}^5)$, where R denotes the radius of the star, the subgiant components of the Algol-type variables form a well-defined linear sequence running approximately parallel to the sequence defined by the dwarfs. It becomes therefore apparent that the subgiant components are not built according to the same stellar model. In other words, the luminosity and the radius are not uniquely determined by the mass. But since $\mathfrak{M}/R$ is related to $L/\mathfrak{M}^5$ according to the empirical results, we can conclude that the luminosity is determined when both $\mathfrak{M}$ and R are given, i.e.

$$L = L(\mathfrak{M}, R), \tag{13}$$

which can also be written parametrically in the following form:

$$L = L(\mathfrak{M}, \lambda), \qquad R = R(\mathfrak{M}, \lambda) \tag{14}$$

as was first suggested by Parenago and Massevich, although they failed to give a physical meaning to the parameter λ thus introduced. It is obvious that the function $L(\mathfrak{M}, R)$ can be determined from Parenago and Massevich's empirical plot.

In order to investigate the physical meaning of λ, Struve (**3**) has first plotted the bolometric absolute magnitudes of the individual components against their masses, and then the departure (in absolute magnitudes) of each star from Kuiper's mass–luminosity curve (**61**) is further plotted against the mass ratio of the two components ($\mathfrak{M}_1/\mathfrak{M}_2$). The second plot is reproduced here in Fig. 5. All stars except two are taken from Parenago's compilation. The two additional stars, XZ Sagittarii (**59**) and DN Orionis (**1**), are denoted by the crosses in the figure. We can clearly see the excellent correlation between the departure from the mass–luminosity relation and the mass ratio. Consequently, we can identify λ in equation (14) as the mass ratio of the two components (**3**).

The binaries we have considered are close ones. When one of the components of such a binary happens to be a subgiant, its surface may come into touch with the inner contact surface of the system. Indeed, F. B. Wood (**62**) found that many subgiants are so large that they approach or perhaps even exceed slightly the limit of stability defined by the inner contact surface. This result is, of course, consistent with the existence of gaseous streams in such systems

established in the previous sections. The subgiant component ejects mass because it has filled up one lobe of the inner contact surface. The size of the contact surface is directly calculable from the mass ratio of the two components, as is illustrated in Fig. 1. J. A. Crawford (**63**) has plotted a curve of the mean radius of the lobe of the inner contact surface (in units of the separation) against $\mu = \mathfrak{M}_2/(\mathfrak{M}_1 + \mathfrak{M}_2)$, and found that the subgiant secondaries whose mass is less than 1 ⊙ all cluster near the curve. Crawford's plot therefore confirms the

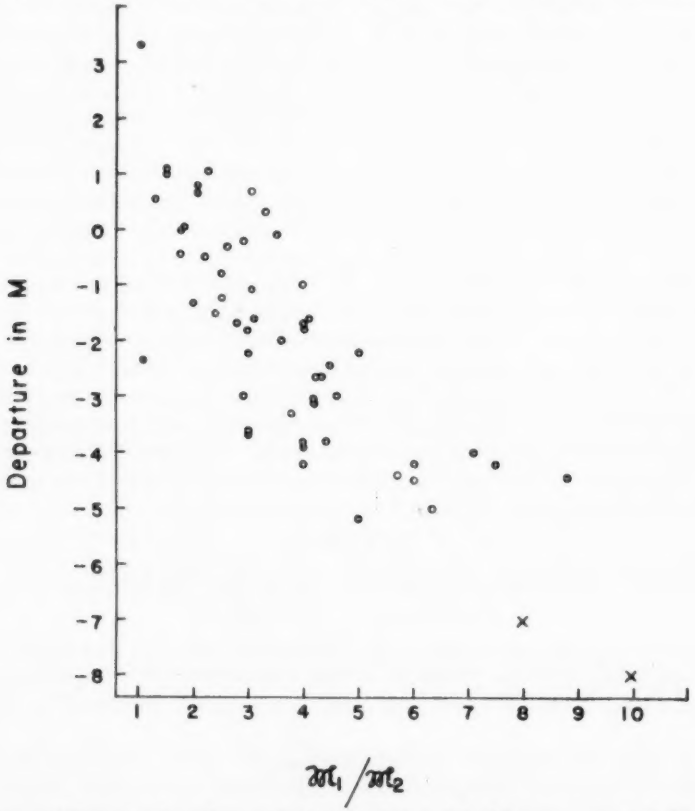


FIG. 5.—*Departure from the mass-luminosity relation of the fainter components of the Algol-type variables. The plot shows empirically that the physical nature of the fainter component of an Algol-type system is dependent not only on its own mass but also on the mass of its companion.*

conclusion of Wood. Hence the mass ratio of the two components plays an important role in determining both the physical nature (such as the departure from the $\mathfrak{M} - L$ relation) as well as the geometrical configuration of the subgiant component of a close binary. From this empirical result we will naturally enquire whether the physical nature of the subgiant component is related to the geometrical restriction. This question leads directly to the problem of the evolution of the subgiant secondaries. Crawford (**63**) suggests that the subgiant component is in the stage of expansion according to the Sandage and Schwarzschild model of stellar evolution (**64**). But after the star comes into touch with the inner contact surface, its size can no longer increase and becomes stationary.

Further expansion of the subgiant component leads to the ejection of mass through the contact surface, which is observed as the gaseous stream in the binary system.

We should point out that the mere fact that we observe, in many close binaries, gaseous streams as well as subgiant components which fill up one lobe of the inner contact surface of the system leads to the conclusion that a star expands instead of contracting during a certain phase of its evolution. For if a star contracts, the gaseous streams would eventually die down and all components should be smaller than their corresponding inner contact surfaces. Thus, the result of investigation of the close binary stars provides independent empirical evidence, aside from the colour–magnitude study of the globular clusters by Mt Wilson astronomers (**65**), of the direction of evolution of the stars from the main sequence.

There remain some difficulties in this otherwise very satisfactory explanation of the subgiant secondaries of the Algol-type variables. According to the interpretation given above, the subgiant secondary should have a greater initial mass than that of the primary so that the secondary was more luminous and evolved faster than the primary. If the primary, which is usually a normal main-sequence A star, corresponding to a mass of about $3\odot$, has not absorbed an appreciable percentage of mass ejected by the secondary, then we have to assume the initial mass of the secondary to be much greater than $3\odot$. Even if the primary has absorbed all mass ejected by the secondary, we still have to assume the initial mass of the secondary to be greater than $1{\cdot}5\odot$. The present masses of the secondary components range from $0{\cdot}2$ to $1{\cdot}0\odot$. Hence in some extreme cases the star has lost more than nine-tenths of its initial mass. This is very difficult to believe. This difficulty is possibly connected with another unexplained fact, namely, that the subgiant components in the eclipsing variables occur predominantly between spectral types F and K.

8. Evolution of the Components of Close Binaries

It has long ago been suggested (**1**) that all Wolf-Rayet stars could be components of close binary stars. They eject matter in a manner similar to that found in the subgiant components of the Algol-type variables, because their atmospheres overflow the inner contact surface. Also, since the discovery of the binary nature of Nova Herculis by M. F. Walker (**66**), it has been further suggested that the novae and nova-like objects may all be components of close binary systems (**67**). Indeed, there are several nova-like objects which have been definitely identified as the components of some binary systems. Consequently, we have proposed some suggestions as regards the origin of the novae and nova-like objects based on the possible binary nature of such stars (**4**).

In Fig. 6 we have roughly drawn the evolutionary track on the H–R diagram of a single star based on the study of globular clusters (**65**) as well as on theoretical computations (**68**). That track is shown by the solid line. The evolutionary track of the component of a close binary which has the same initial mass as the single star represented in the diagram is difficult to obtain because of the continuous loss of mass. Immediately after the component comes in touch with the inner contact surface of the dynamical system, the star will follow approximately a sequence of constant radius since the size of the inner contact surface changes very slowly with the mass ratio. The exact evolutionary track

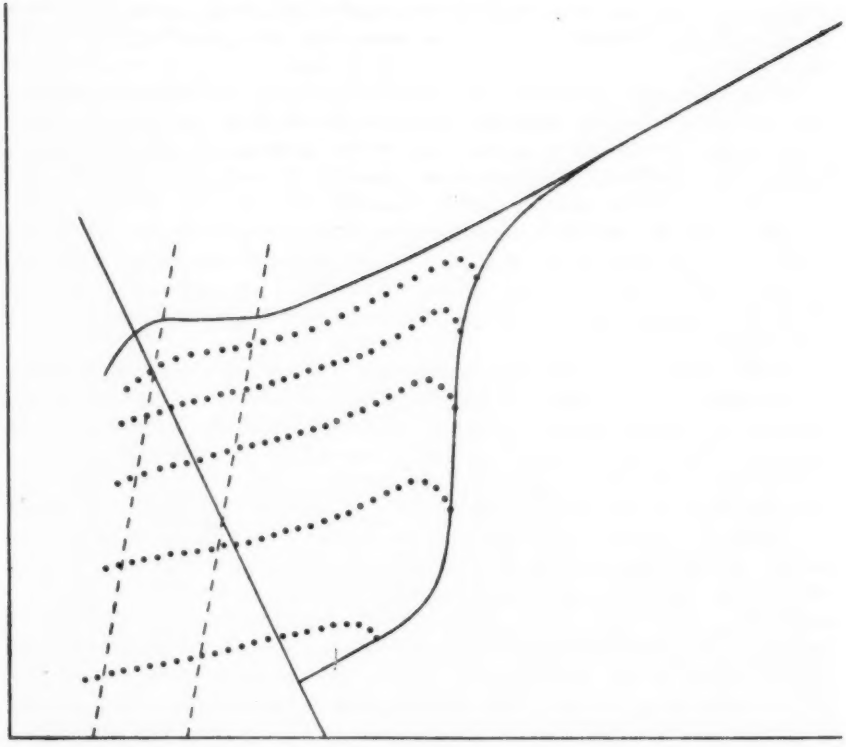


FIG. 6.—*A suggested region of unstable stars on the H-R diagram. Different dotted curves show the evolutionary tracks on the H-R diagram of stars which are the components of close binary systems of different separations. The unstable zone is located between the two broken lines in the figure.*

on the H–R diagram, however, cannot be obtained without detailed calculation. We believe that the star, after having touched the inner contact surface, will move towards the upper left on the H–R diagram. Our reason for this prediction is that the energy production underneath the envelope of the star will not be immediately affected by the loss of mass on the surface. In other words, an increasing amount of energy will be radiated away by a surface of the constant size. As a result, both the effective temperature and luminosity increase. When the envelope no longer expands, the star will evolve according to a track of constant mass, and the star will turn towards the left in the H–R diagram. The evolution at this stage will be more or less like that of a single star of a mass less than the initial mass of the component. At this stage the internal structure of stars of different masses may be regarded as built on the same model except perhaps their envelopes. A mass–luminosity relation, therefore, will be expected, at least roughly. Therefore, the positions on the H–R diagram of stars of smaller mass should lie below that of larger ones. From this heuristic reasoning we draw in Fig. 6 the dotted lines, each of which represents qualitatively the evolutionary track of the component of a close binary system. We have assumed that all these tracks represent stars of the same initial mass. Hence the component

belonging to a binary system of comparatively large separation will have a greater final mass than one in a close system, because the total loss of mass is smaller in the former than in the latter. Consequently, running down the diagram, different tracks denote decreasing separations of the binary systems.

Now we can describe our hypothesis concerning the existence of a sequence of unstable stars which can be specified by one single parameter (for a given initial mass, of course), i.e. the total mass lost during the earlier stage of evolution. The instability which produced pulsation at a certain stage of evolution in cluster-type variables exists in every evolutionary track in Fig. 6. The instability, originated in the central core, manifests itself in different ways on different tracks for a reason which will be discussed later. For a single star which has no companion and has lost no mass, the instability produces pulsation. The component of a not too close binary will evolve naturally, too. When it reaches the zone of unstable stars it will become either an RR Lyrae star or one slightly below it; the latter should behave like an RR Lyrae star. In accordance with C. R. Lynds' recent investigation (**69**), we believe that UX Monocerotis is representative of the latter case. But when the star is a component of a very close binary system and has lost a great deal of mass, the instability shows up in a violent manner, causing a nova-like outburst. We include such stars as SS Cygni variables as intermediate objects in the sequence. We have drawn two straight broken lines in Fig. 6 which enclose this hypothetical sequence of objects. From this hypothesis several results can be derived and be compared with observation.

Novae and SS Cygni variables should be components of close binary systems. An eclipse can be seen only if the plane of the binary orbit lies nearly in the line of sight, say within 10°. Five out of six binaries would, on the average, fall outside this limiting inclination. In this way one can explain (**67**) the reason why among several ex-novae only one has thus far been actually observed to be an eclipsing binary.

A single star which once becomes a cluster-type variable will not pass through the stage of a nova. Therefore the frequency of occurrence of the cluster-type variables in a group and that of the novae in the same group are not related. This agrees with the observed result.

Novae, SS Cygni stars, and cluster-type cepheids of the same initial mass take, according to the present theory, different times to evolve before reaching the same stage characterized by their instability: a long time for a star on the main sequence to become an RR Lyrae star, but a comparatively shorter time for the same dwarf to become a nova. Thus, the nova is a young star compared with the RR Lyrae star, but its position on the H–R diagram makes it appear as older than the latter. The subdwarf nature of the ex-novae has been convincingly shown by D. B. McLaughlin (**70**) who finds, from the temperature and the bolometric absolute magnitude, the radius of the ex-nova to be around 0·14 solar radius. McLaughlin's prediction is confirmed by the recent investigation of Walker (**66**).

The mass should decrease statistically from RR Lyrae stars, to SS Cygni stars, and finally to novae. Novae could have very small masses, because they have lost a great part of their initial mass. The scanty observational evidence that we have about the masses of various objects considered here, seems to agree with our prediction. The recent photometric study of the eclipsing system of

Nova Herculis by Walker (**66**) indicates extremely small values of both its mass and the separation. AE Aquarii is another star belonging to the unstable sequence. Its light variation would place it between the novae and the RR Lyrae stars. The mass of the exploding B component determined by Joy is of the order of $1 \odot$ (**71**).

After the binary nature of Nova Herculis was discovered, W. Baade (**66**) pointed out that if we trace back the history of Nova Herculis 10^9 years or so, to the red-giant stage, the other component of the binary would have revolved inside the giant star, provided the orbit has not been changed since then. This is, of course, inconceivable. According to our hypothesis the nova has never been a red giant.

Let us now consider the question: why does instability in the core generate in some stars mild pulsations, while in others it provokes violent outbursts of various degrees? We suggest the following explanation, based on the idea of damping by the envelope. A cluster-type cepheid or a Population II cepheid has, above its unstable core, a sizable envelope which damps the instability. Mild pulsation will be the result of damped outflow of excess energy generated in the core, but mild pulsation does not carry away enough excess energy to remove the instability. Consequently, the instability persists and the star pulsates according to its natural frequency. A nova, on the other hand, has a very thin envelope because its mass has been continuously chopped off at the outermost layers of its atmosphere, although its central core may not be too different from that of a cluster-type cepheid. Hence, an instability in the core is able to throw its atmosphere high up into space, causing a nova explosion. The excess energy being completely relieved by the explosion, the star becomes temporarily stable, until another instability is developed inside the core. According to this hypothesis, both the pulsation of cluster-type (Population II) cepheids and the outburst of novae (or nova-like objects) are regarded as a means of relieving the excess energy created by instability inside the star.

Our picture finds support in the velocity-curves of RR Lyrae stars. Being discontinuous, they start a new branch near the minimum velocity before the preceding one ends near maximum velocity. Struve (**72**) suggested that the so-called pulsation may be a kind of successive formation of shells in the star. Before the first shell returns to the star, another shell is formed and thrown out. We may regard the shell formation as the result of a mild outburst. Later on, it was found that W Virginis shows a velocity-curve similar to that of RR Lyrae (**73**).

Finally, let us consider the binary system AE Aquarii. The nova-like component of this system has passed its mass-losing stage and has reached the zone of instability shown in Fig. 6, but its companion, according to a recent study by J. A. Crawford and R. P. Kraft (**51**), is now in the stage of expansion. The companion is now losing mass, a part of which falls into the nova-like component. They have estimated the order of magnitude of the rate of this transfer of mass and suggested that the explosion is the result of mass which has fallen into the atmosphere of the nova-like component. In accordance with our hypothesis, such a collection of mass will not affect the stars' explosive nature, because the outburst is determined by the instability in the central core. We believe that losing or gaining mass in the atmosphere has no short-range effect on the evolution of the star. But because of the mass collected in the envelope,

the time spent in the exploding stage of the nova-like component could be lengthened. In other words, even without the process of mass collection, the nova-like component of AE Aquarii would still show intermittent outbursts.

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